設計為本的教師素養教學專業學習理論與實踐

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老師的心

準備最好的內容

引導最適的學習方式

生生都享受學習的樂趣

力，何以致之!

「設計為本」的教師專業發展進路

設計為本的數學教師教育

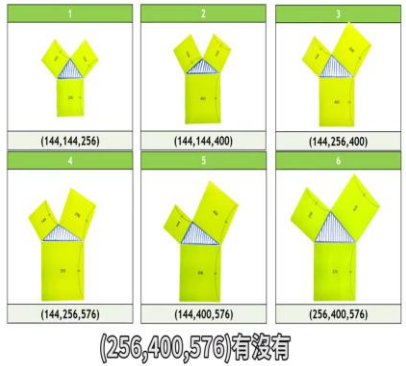
理念：教師是研究者

壹、設計思維

- 例1. 畢氏定理（賴姵玲，2020， https://youtu.be/YNgirVVDipo ）
- 議題：「有趣、有感、有想法」的論證學習進路?如何設計？
- 設計例：歸納法學畢式定理



拼板遊戲與畢氏定理（畢氏定理的發現）（數學奧基進教室，109年）



拼板遊戲與畢氏定理（畢氏定理的發現）（數學奧基進教室，109年）

設計思維

- 目標：「有趣，有感，有自己想法」滋潤這學習進路
- 設計理論：數學的三個世界（D. Tall ，秦爾聰）之「Embodied」
- 設計原則：目標即原則

  引導式發現（主動思考）
- 設計策略：推理方法(演繹、歸納、溯因)的轉換（演繹轉歸納）
- 教學實作：參閱拼版遊戲與畢氏定理影片（賴姵玲，2020， https://youtu.be/YNgirVVDipo ）

設計思維

討論：

- 1. 探究式論證的另例： a+b 幾何作圖的一維，二維類推
- 2. 畢氏定理與三一律的連結（空間推理）
- 3. 畢氏定理的取信式證明很多，是擴張個人數學論證空間的好材料

壹、設計思維

- 例2. 溯因推理(若P則Q，已知Q，找P) 偵四物語（巫佳錚，2019， https://youtu.be/0TxgeIYahmc）
- 四邊形對角線：溯因推理



偵四物語（四邊形對角線：溯因推理）（數學奠基進教室，108年）



設計思維

議題：特殊四邊形內容的學習定位

- Van Hiele 的幾何思維模式發展的好材料
- 包含關係的非形式演繹推理（Van Hiele）
- 設計理論：Van Hiele幾何思維模式的發展（層次2的實作）
- 設計策略：可嘗試的具體操作（做說連結）
- 語言演化（調整）

設計原則：

討論：

壹、設計思維

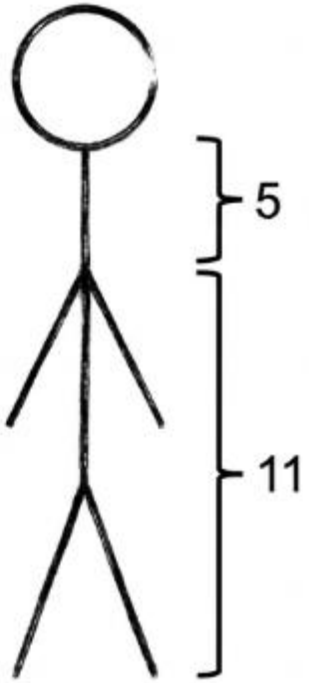
- 例3. 比例推理的診斷教學（林福來等，1986）
- 現象：加法策略跨年級普遍性
- 議題：認知發展階段的必然 vs.可教學改進
- 分析現象分析： Adders’的解題策略分析，Taiwan為Non-adders
- 溯因探究：(現象—假設—檢驗—歸因)

診斷教學設計思維

- 設計理論：認知的平衡模式(Piaget)
- 設計原則：認知衝突(如何自主觸動)；有感、有想法
- 設計策略：（脖子，身長）(5,11)(6,__)(__,6)
- 脖子不見了！
- 重新學習的進路：動手作圖，直觀辨識加法策略、乘法策略的圖形變化差異

【現象痛點】
學生普遍使用錯誤的「加法策略（Adders）」
（例如：5:11 推導出 6:12）

【理論與原則】
皮亞傑認知平衡模式 → 引發內部「認知衝突」



若身長變為6，依加法策略脖子變多少？
(6-6＝0)

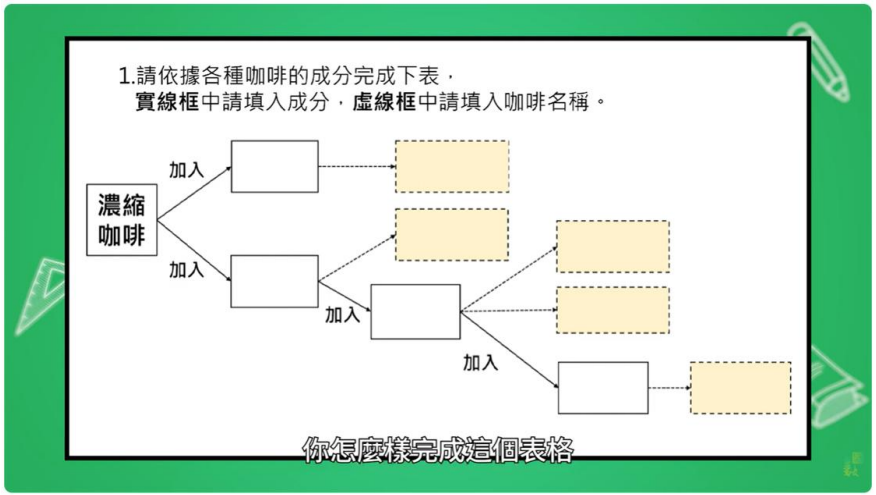
脖子不見了！

用視覺直觀戳破加法策略的謬誤，迫使學生重建乘法策略。

壹、設計思維

- 例4.「比例與濃度」的系統性思維學習進路

  (劉建成等，2023， https://youtu.be/TmC3uzIOLv8)



系統性思維促進比的概念與素養（比例與濃度）〔數學素養微課程，112年〕

設計思維

- 理論：建立具發展性的系統性思維框架？

(劉建成等，2023， https://youtu.be/TmC3uzIOLv8)



系統性思維促進比的概念與素養（比例與濃度）〔數學素養微課程，112年〕

壹、設計思維

- 例5. 比例推理：固定單價法 VS. 多元推理策略
- 固定單價法（Gold， 1980）為教學策略
- 設計目標：學習成就，自信心
- 設計原則：減低認知負荷
- 設計理論：認知資訊處理理論
- 設計策略：教學實驗檢測
- 新的挑戰：以多元推理策略為學習目標，如何研發？

壹、設計思維

- 例6. 數學是一種符號科學
- 議題：何謂符號的了解？
- 符號的表層與深層結構意義(R. Skemp)
- 雞蛋的密碼，我懂你!（胡詩菁，2024，https://youtu.be/Oxo459v-QyE）
- 個體經驗的符號意義
- 詮釋符號的根源



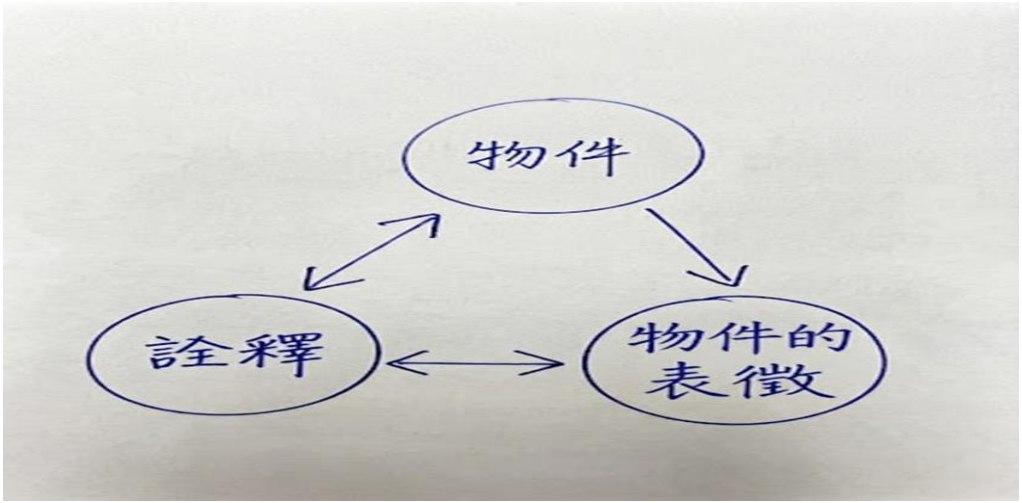
符號的意義

- 2甲+3丁是什麼？物件 VS. 價錢
- 甲+ 5=丙，甲+ 7等於____
- 甲+乙+丙=甲+丁+丙，你認為

  一定成立，一定不成立，不一定
- e+f=8 ， e+f+g=____
- 比較 n^2 與 2n 的大小

符號科學

- 表徵內換算
- 表徵轉換



符號科學

- 例7. 表徵對對碰（巫佳錚.2021, https://youtu.be/ehzJ2a9_Zds)



理論：表徵理論，符號科學

策略：

- 分類活動
- 命名活動

原則：有趣，有感，有自己想法

壹、設計思維

- 例8. 建模(個別化的建模)：跑多快（侯雪卿.2024, https://youtu.be/nBMU5PUsgnA)



速率的情境連結－微型活動3：速率的探究：我跑多快？



速率的情境連結－微型活動3：速率的探究：我跑多快？

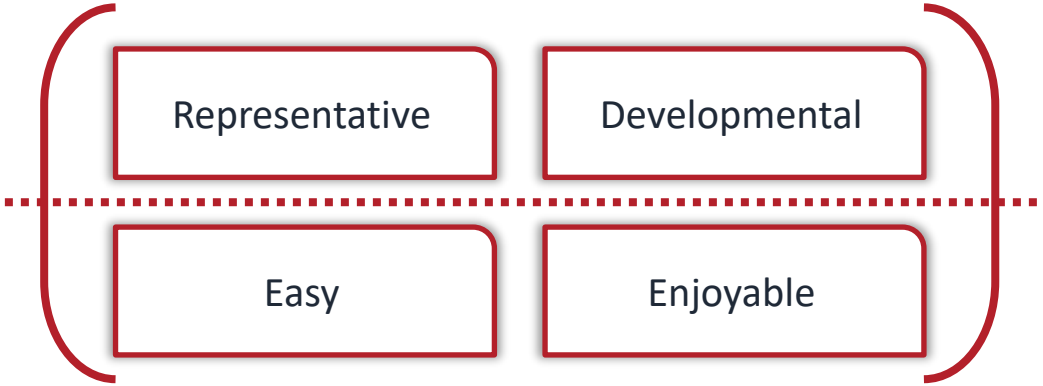
貳、教師在設計為本的數教環境中的學習：JDM為例

- 學什麼？
- 怎麼學？

Just Do Math

I. Design “WHAT”?

Generic learning activity (GLA):



For learning (Mason & Pimm, 1984)

For instruction (Cheng, 2000)

Fundamental Principles: Interesting & Having Sense

I. Design “WHAT”?

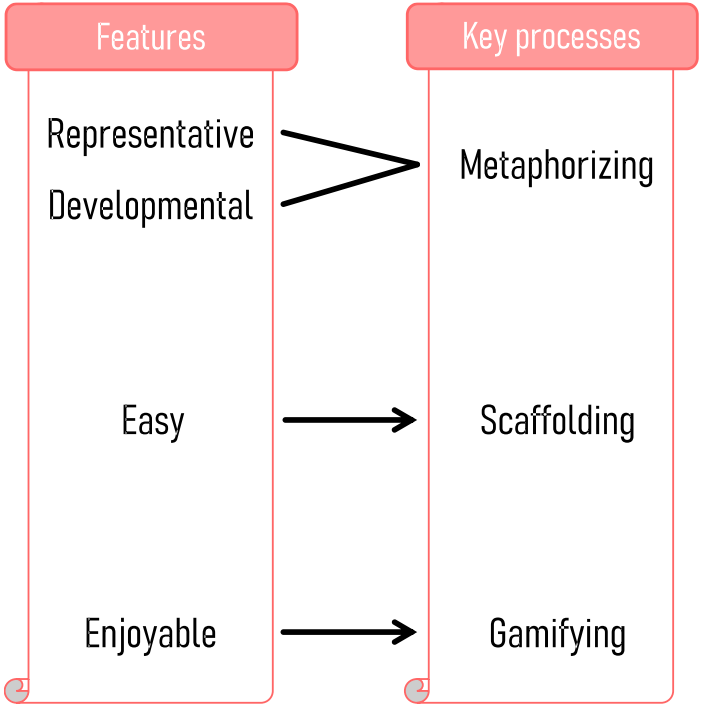
MGAs, as generic learning activity, aim at…

1. Developing fundamental prerequisite mathematical ideas before they learn in regular lessons .
2. Constructing references for mathematical concepts through the operation of manipulable representations.
3. Increasing students’ learning motivation through gamified activity.

Wang, T. Y., Lin, F. L., & Yang, K. L. (2021). Success factors for a national problem-driven program aimed at enhancing affective performance in mathematics learning. ZDM–Mathematics Education, 53(5), 1121-1136.

I. Design “WHAT”?

The framework of designing MGA:



Steps

- ① recognizing what the target domain of an intended mathematical concept is. (e.g. 1, prime, composite number)
- ② recognizing a source domain to embody the target domain which could be inferred as conceptual metaphors of the source domain. (e.g. points build rectangles)

- ① providing tools which can be manipulated to generate examples. (e.g. Point)
- ② providing goals which can trigger learners’ awareness of the insufficiency of their emergent conceptions and enact the development of conceptual metaphors. (e.g. Diagnostic and provocative intervention)

- ① generating multiple task processes and outcomes. (e.g. The records in the game playing, the naming of the classification)
- ② generating relevant feedback as tasks evolve. (e.g. Designed worksheets)

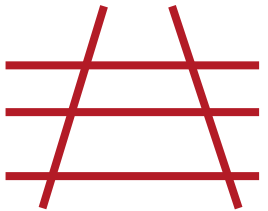
I. Design “WHAT”?

The entry of designing :

1. Problematic analysis :

- Choose topics to reflect the problematic
- Problematic due to

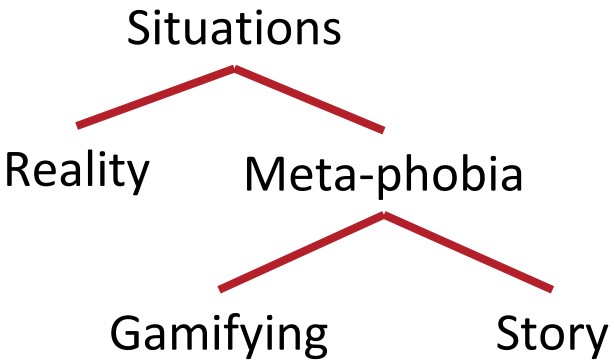
$$\begin{cases} Inappropriateness\ of\ the\ teaching\ material, eg. \\ Teaching\ difficulties, eg. function \\ Learning\ difficulties, eg. prime\ number \\ Integrated\ with\ ICT, eg. square\ root\ number \\ \dots \end{cases}$$



I. Design “WHAT”?

2. Analysis with the conceptual Field : (Vergnaud, 1983, 2009)

{ S , I , R }



Invariant

Representation

Multiple representations

- Natural language
- Enact
- Iconic
- Symbolic

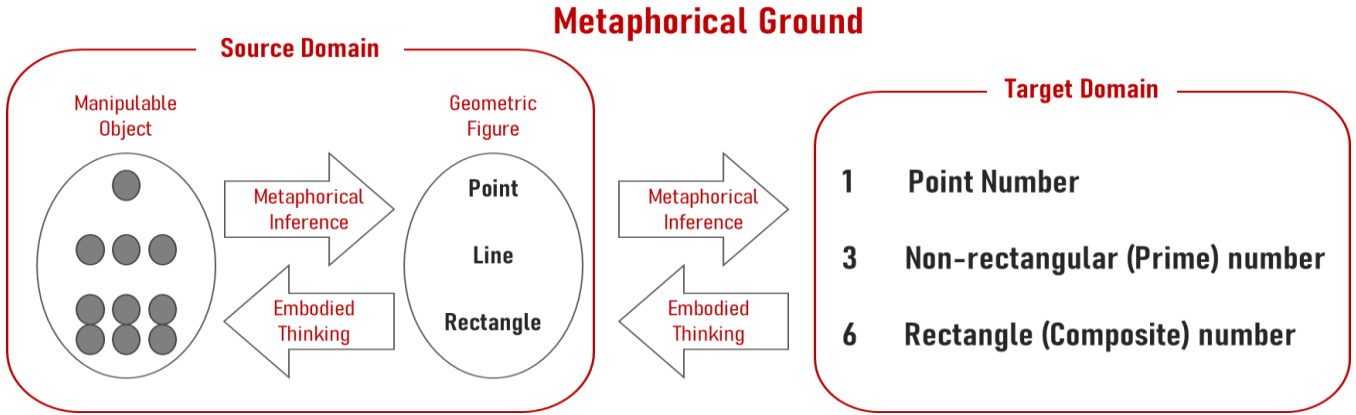
Example: Prime Number

- Topic : Prime Number
- Problematic analysis :
  1. The formal definition is too abstract
  2. Realistic situation is not available
- Situation : Meta-phobia – Gamifying
- Invariant : No proper factors

- Representation :

Scaffolding
- Factorizing → multiplication
- → formula of rectangle area as representation of multiplication
- → Manipulable
- → Gamifying

Entitled with rectangular number game
Initial designer: R. R. Skemp, 1983



Design tasks for understanding, towards competencies (Ruthven et al., 2009)

Grand theory : enactivism

Designing principles (Intermediate theory) (addressed by facilitator)

- Intrinsic motivation
- Sustaining “a sense of”
- Diagnostic
- Language evolution

Design tasks for understanding, towards competencies (Ruthven et al., 2009)

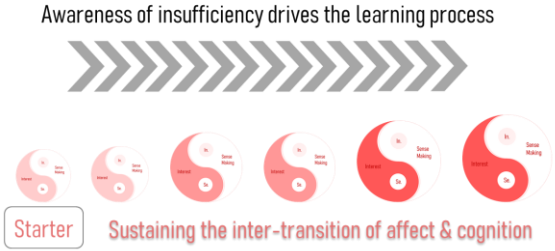
Design tools/strategies : (exercised by designer in JDM)

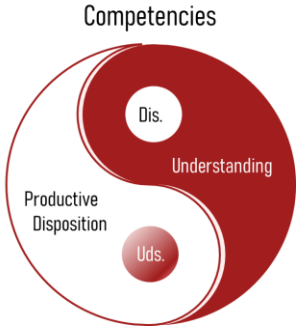
- Defining
- Classifying (cognitive or affective)
- Comparing
- Naming
- Grouping
- Matching
- Beyond calculation, toward ”conceptual” reasoning
- Shifting reasoning modes (induction, deduction, abduction)
- Applying the tri-elements with respect to each 21st C. thinking skills
- Crucial questioning
- Isomorphic transformation
- …

Analogy pervading all of learning issues (類推，一以貫之)

II. Students' learning

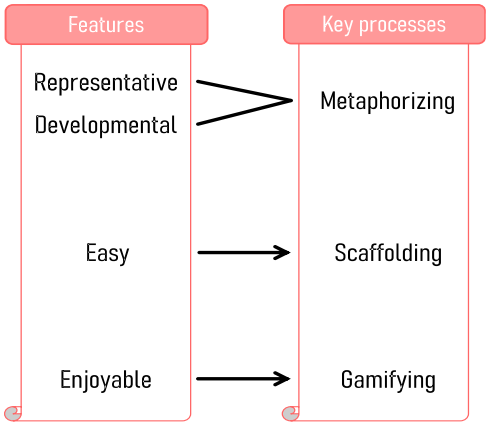
- What to learn?
- How to learn?





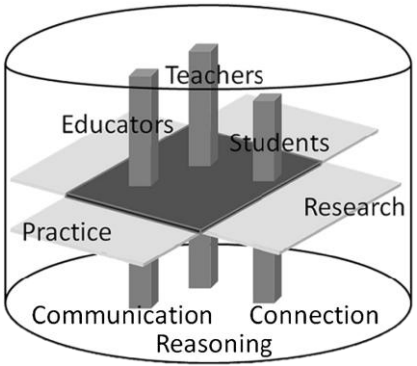
III. Teachers' learning

- What to learn?
- How to learn?



IV. Educator-Researchers' learning

- What to learn?
- How to learn?

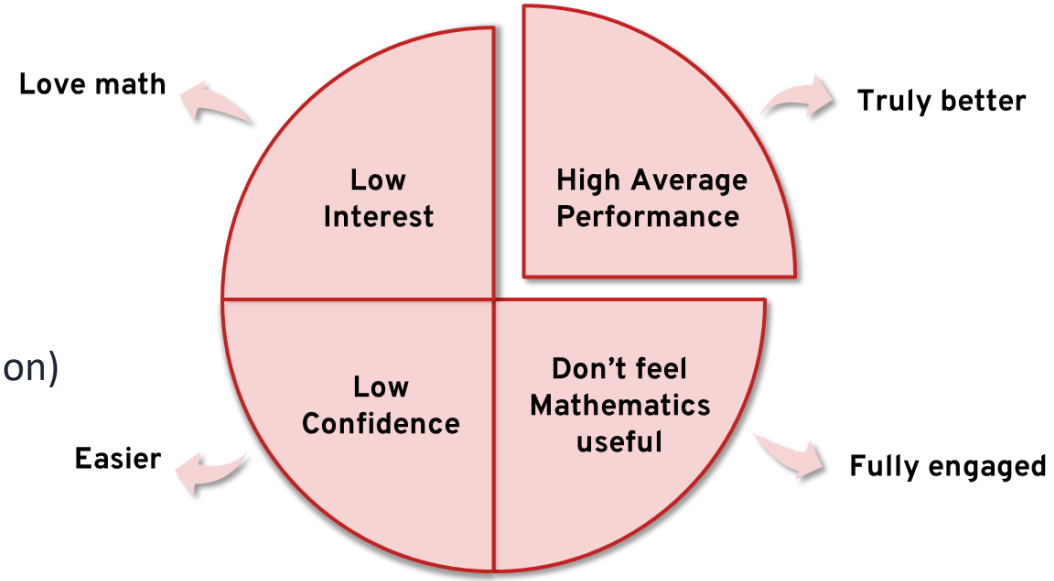


02 | Students’ learning

II. Students' learning

What to learn?

- Engagement
  - Cognitive
  - Affective
  - Behavioral (Teacher's observation)



Lin, F. L., Wang, T. Y., & Yang, K. L. (2018). Description and Evaluation of a Large-Scale Project to Facilitate Student Engagement in Learning Mathematics. Studies in Educational Evaluation, 58, 178-186.

Lin, F. L., Wang, T. Y., & Chang, Y. P. (2018). Effects of large-scale studies on mathematics education policy in Taiwan through the lens of societal and cultural characteristics. ZDM–Mathematics Education, 50, 587-600.

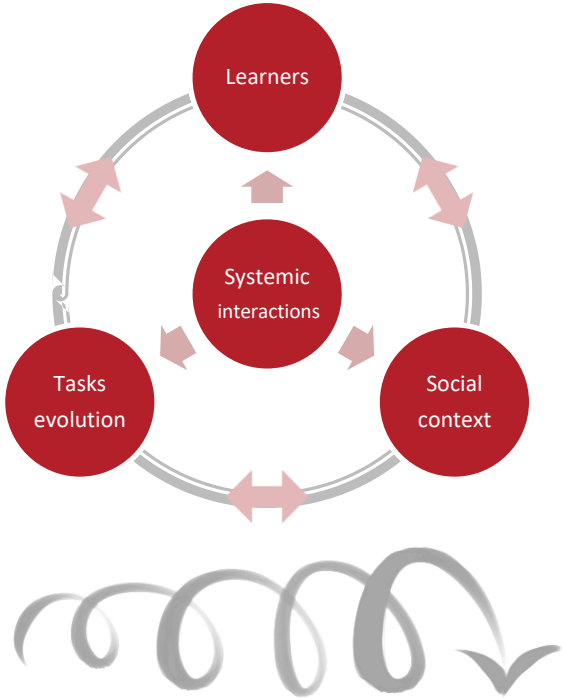
Lin, F. –L. (2018, May). Mathematics Education Reform in Taiwan：Mathematics Grounding Activity as a Driving Force. Plenary Speech at the 8th ICMI-East Asia Regional Conferences on Mathematics Education, Taipei, Taiwan

II. Students' learning

How to learn?

- The learning theory of MGA learning environment
  - Enactivist perspective :

    Systemic interactions between learners, tasks evolution and social context



Wang T.Y., Lin F.L., Yang K.L. (2021). Success factors for a national problem-driven program aimed at enhancing affective performance in mathematics learning. ZDM–Mathematics Education, 53(5), 1121-1136.

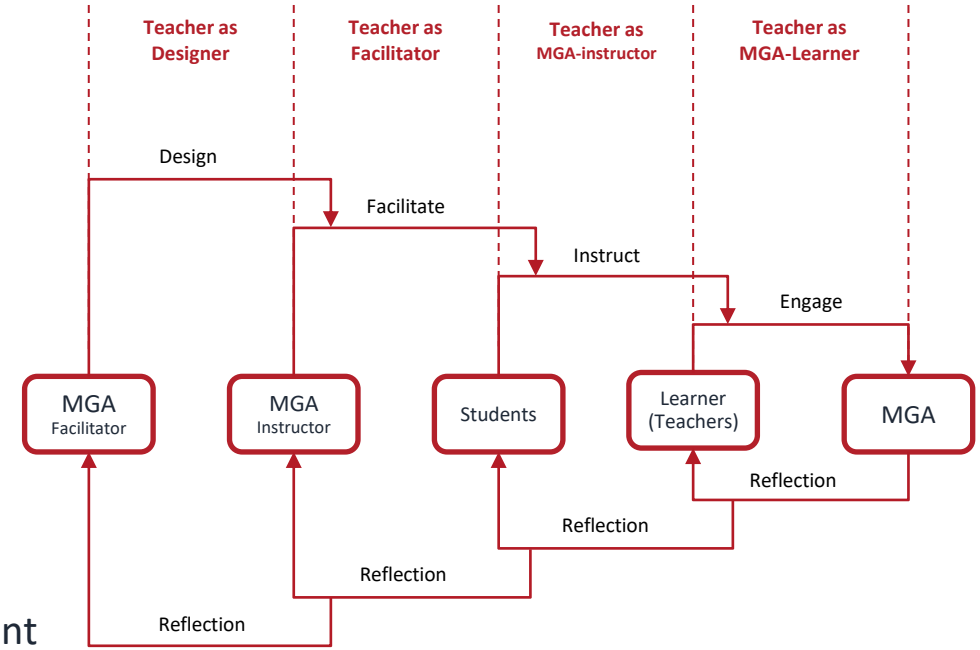
Yang, K. L., Lin, F. L., & Tso, T. Y. (2021). An approach to enactivist perspective on learning: Mathematics-grounding activities. The Asia-Pacific Education Researcher, 1-10.

03 | Teachers’ learning

III. Teachers' learning

What to learn?

1. Teacher as MGA-learner
   - Awareness of a new teaching approach
   - Awareness of students' interesting
2. Teacher as MGA-instructor
   - Practicing MGA, a new approach (teacher-center strategies to student-center strategies)
3. Teacher as Facilitator
   - Communicating teaching
   - Pedagogical communicator
4. Teacher as Designer
   - Diagnostic teaching
   - Becoming an "expert" teacher, consultant textbook editor

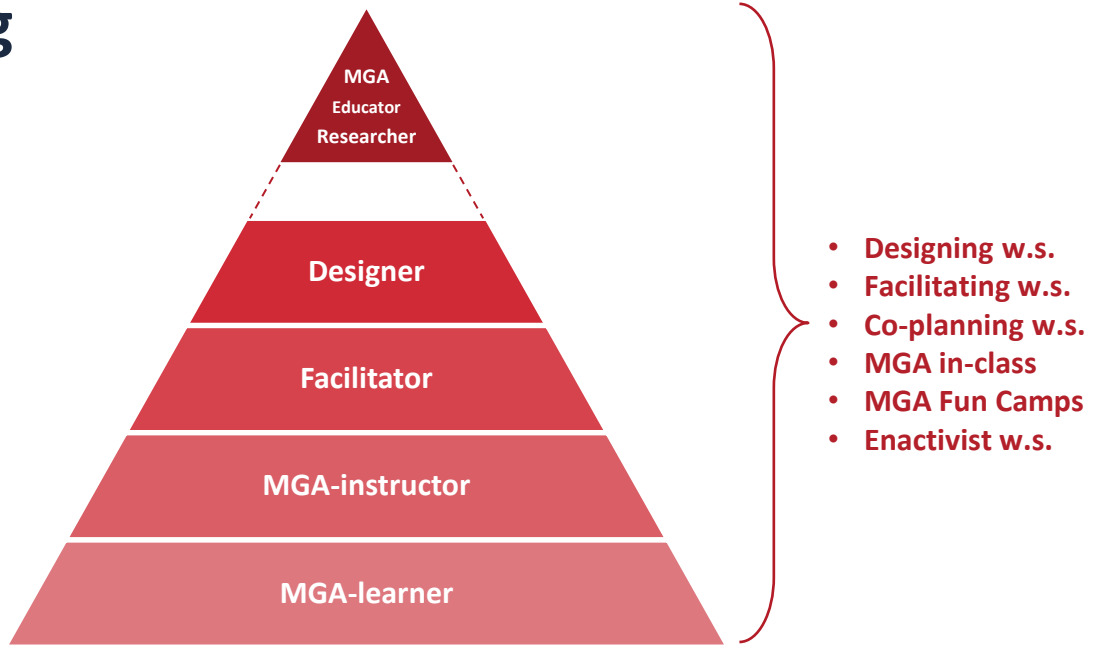


III. Teachers' learning

How to learn?

- Four-stage, transformative hierarchical approach

層疊式



Lin, F.-L., Yang, K.-L., & Wang, T.-Y. (2016, July). Transformative cascade model for Mathematics teacher professional development. Paper presented at the 13th International Congress on Mathematical Education, Hamburg, Germany.

Chang, Y.-P., Lin, F.-L., & Yang, K.-L. (2020, February). A Mathematics Teacher's Learning in Design-based Research: The Brokering Supports through Different Collaborative Groups. Paper presented at ICMI Study 25, Lisbon, Portugal

III. Teachers' learning : A specific case

What to learn?

- The principles of designing conjecturing tasks (Lin, Yang, Lee, Tabach & Stylianides, 2012)
  - Goal oriented observation
  - Construction
  - Transformation
  - Reflection

How to learn?

- Lee, E. J., Lee, K. H., & Park, M. (2019). Developing preservice teachers' abilities to modify mathematical tasks: Using noticing-oriented activities. International Journal of Science and Mathematics Education, 17, 965-985.

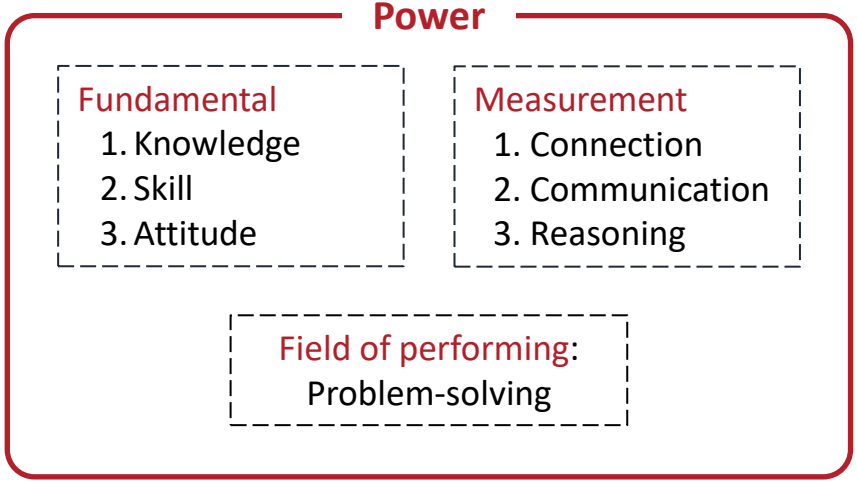
Lin, F. L., Yang, K. L., Lee, K. H., Tabach, M., & Stylianides, G. (2012). Principles of task design for conjecturing and proving. In G. Hanna & M. de Villiers (Eds.), Proof and proving in mathematics education: The 19th ICMI study (pp. 305–326). New York, NY: Springer.

04 | Educator-Researchers' learning

IV. Educator-Researchers' learning

Learning Theory

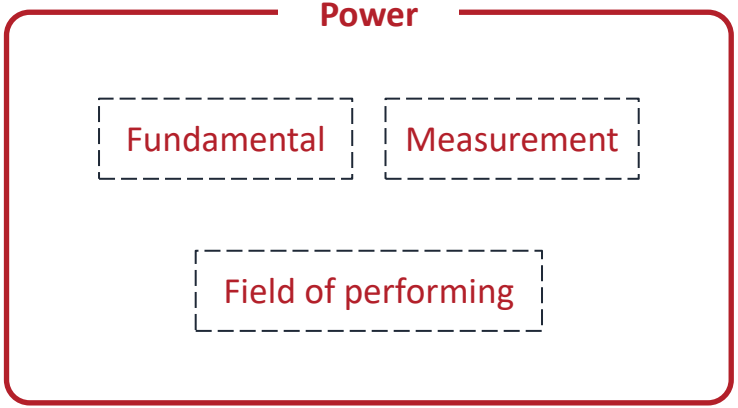
- Students' mathematical power
- Teachers' pedagogical power
- Educators' educative power

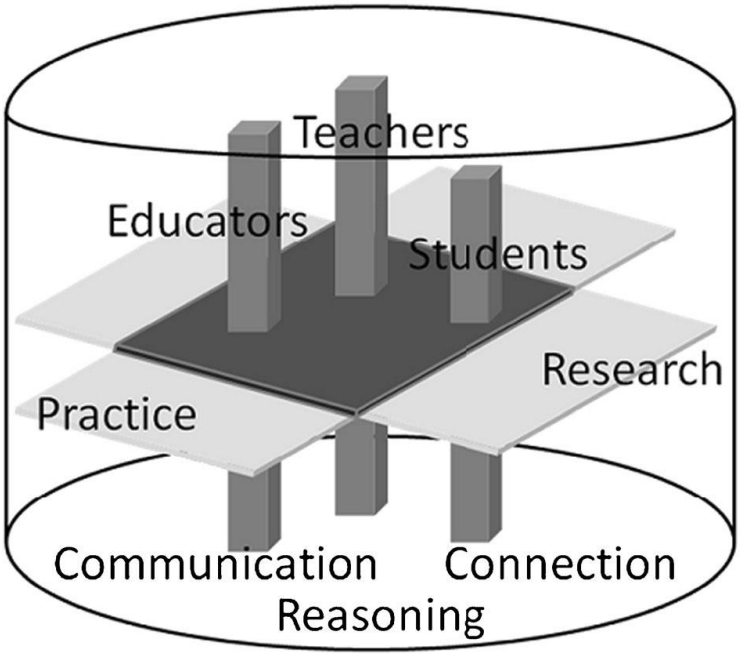


Yang, K. L., Hsu, H. Y., Lin, F. L., Chen, J. C., & Cheng, Y. H. (2015). Exploring the Educative Power of an Experienced Mathematics Teacher Educator-Researcher. ESM, 89(1), 19-39.

IV. Educator-Researchers' learning

$$\begin{pmatrix} Mathematical \\ pedagogical \\ educative \end{pmatrix}$$ Power





Yang, K. L., Hsu, H. Y., Lin, F. L., Chen, J. C., & Cheng, Y. H. (2015). Exploring the Educative Power of an Experienced Mathematics Teacher Educator-Researcher. ESM, 89(1), 19-39.

IV. Educator-Researchers' learning

What to learn?

- About designed tasks
  1. Criteria of evaluating products
  2. Theory in practice
- About TPD
  1. Approaches of facilitating workshop
  2. Theory in practice
- About tasks designing
  1. Specific design strategy
     - Transformative
     - In particular to in general
  2. Theory in practice
- About the development of competencies in educative power, such as reasoning, communication, connection
  1. Among the variety of principles of instruction & learning, how to argue and optimize a simple one?
  2. Theory in practice

Yang, K. L., Hsu, H. Y., Lin, F. L., Chen, J. C., & Cheng, Y. H. (2015). Exploring the Educative Power of an Experienced Mathematics Teacher Educator-Researcher. ESM, 89(1), 19-39.

IV. Educator-Researchers' learning

How to learn?

- Self-learning on exercising new approach (ref. activity theory) on facilitating TPD
- Dialogue between educator-researchers based on elements in cascading model: goal, theory, structure, content
- Dialogue during the workshop among teachers and educator-researchers through the elaboration of competencies, reasoning, connection and communication, in educative power
- Analysis, induction and elaboration during the evaluation of teachers' product
- Journal paper writing on reporting the designing workshop
- Evaluating the program internally and externally

Chen, J. C., Lin, F. L., & Yang, K. L. (2017). A Novice Mathematics Teacher Educator-Researcher's Evolution of Tools Designed for In-Service Mathematics Teachers' Professional Development. JMTE, 1-23.

Lin, F. L., Yang, K. L., Hsu, H. Y., & Chen, J. C. (2018). Mathematics teacher educator-researchers' perspectives on the use of theory in facilitating teacher growth. Educational Studies in Mathematics, 98(2), 197-214.

IV. Educator-Researchers' learning : A specific case

How to learn?

- Teaching video as a means of pedagogical communication

How to learn?

- Editing and producing teaching video

(*) Elementary School Teaching Video List → https://www.ime.ntnu.edu.tw/index.php/en/en-v-e/
(*) Junior High School Teaching Video List → https://www.ime.ntnu.edu.tw/index.php/en/en-v-j/

conclusion

conclusion

Research issues are

everywhere

for all learners

- **Students**
- **Mathematics teachers**
- **Educator-researchers**

under the settings of designing workshops.